

The effect of gluon condensate on holographic heavy quark potential

Youngman Kim*

*Asia Pacific Center for Theoretical Physics and Department of Physics,
Pohang University of Science and Technology,
Pohang, Gyeongbuk 790-784, Korea and
School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea*

Bum-Hoon Lee[†]

*Department of Physics, Sogang University, Seoul 121-742, Korea and
CQeST, Sogang University, Seoul 121-742, Korea*

Chanyong Park[‡]

*CQeST, Sogang University, Seoul 121-742, Korea and
Division of Interdisciplinary Mathematics,
National Institute for Mathematical Sciences, Daejeon 305-340, Korea*

Sang-Jin Sin[§]

*Department of physics, BK21 Program Division,
Hanyang University, Seoul 133-791, Korea*

Abstract

The gluon condensate is very sensitive to the QCD deconfinement transition since its value changes drastically with the deconfinement transition. We calculate the gluon condensate dependence of the heavy quark potential in AdS/CFT to study how the property of the heavy quarkonium is affected by a relic of the deconfinement transition. We observe that the heavy quark potential becomes deeper as the value of the gluon condensate decreases. We interpret this as a dropping of the heavy quarkonium mass just above the deconfinement transition. We finally argue that dropping gluon condensate and pure thermal effect are competing each other in the physics of heavy quarkonium at high temperature.

*Electronic address: ykim@apctp.org

†Electronic address: bhl@sogang.ac.kr

‡Electronic address: cyong21@sogang.ac.kr

§Electronic address: sjsin@hanyang.ac.kr

I. INTRODUCTION

The heavy quarkonium is a good object to study the nonperturbative nature of QCD, see [1] for a review. At zero temperature, for example, the charmonium spectrum reveals important information about confinement and inter-quark potentials in QCD[2]. The heavy quarkonium is one of main probes that provides us with information about thermal properties of the quark-gluon plasma (QGP), see, for instance, [3, 4]. Lattice calculations indicate that the charmonium states will remain bound up to about 1.6 to 2 times the critical temperature T_c of the deconfinement transition[5, 6]. Recent studies based on a QCD sum rule approach have claimed that the change in the properties of heavy quarkonia around T_c could *effectively* be an order parameter of the deconfinement transition [7]. The claim is based on the following observation. At T_c , the energy density ϵ and pressure P of the QCD system increase drastically, which is closely related to a drop of the thermal gluon condensate $G^2(T)$ since

$$G^2(T) \approx G^2(0) - (\epsilon - 3P). \quad (1)$$

For instance, lattice results on the gluon condensate at finite temperature [8] indicate that the value of the gluon condensate shows a drastic change around T_c regardless of the number of quark flavor. The change in the gluon condensate leads to a dropping heavy quarkonium mass around T_c when we ignore shift in the width of the quarkonium [7]. Note that in [7], the mass drops significantly at or just above T_c . From these studies [7, 8], we see that the gluon condensate is very sensitive to the deconfinement transition of QCD and could serve as a messenger for deconfinement transition, which could be observed through the heavy quarkonium in relativistic heavy ion collisions (RHIC). We note here that dropping heavy quarkonium mass at T_c has previously been observed in a study based on a AdS/QCD model. In the study [9] based on the soft wall model [10], the dropping is due to a Hawking-Page transition, and so deconfinement transition, which is qualitatively consistent with QCD sum rule results. We remark here that although the deconfinement transition is driven by the temperature, the drastic change in the value of the gluon condensate is mostly due to the deconfinement transition itself since the value of the temperature does not change much near the transition point.

In this work we study the effect of the gluon condensate on a heavy-quark potential in AdS/CFT [11]. To this end, we adopt a deformed AdS background with back-reaction due

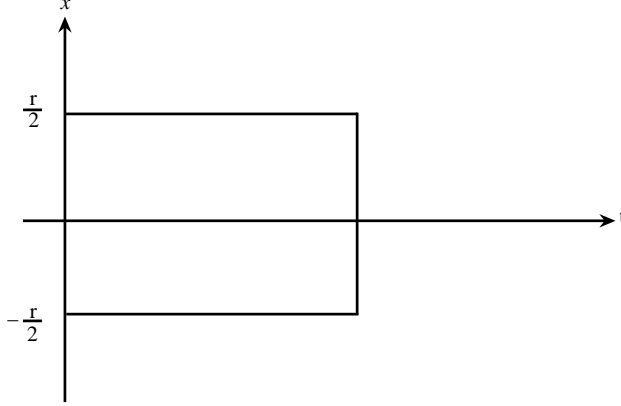


FIG. 1: A Wilson loop.

to the gluon condensate [12, 13, 14, 15]. For a deformed AdS with a non-trivial dilaton potential, we refer to, *e.g.*, [16]. We observe that as the gluon condensate decreases in deconfined phase, the potential becomes deeper. Since the mass of a heavy quarkonium m_{QQ} is roughly $m_{QQ} \approx 2m_Q + V_{QQ} + \text{kinetic energy}$, we could associate the deepening of the potential with dropping of the heavy quarkonium mass. To see this more clearly, we solve Schrödinger equation with our heavy quark potential in the framework of potential model approach. Our observation could be interpreted as the dropping of the heavy quarkonium mass right after the deconfinement transition, which may be a reminiscence of the deconfinement transition since the value of the gluon condensate decreases much with the deconfinement transition.

II. HOLOGRAPHIC HEAVY QUARK POTENTIAL

Through the AdS/CFT correspondence, we can easily evaluate the potential between heavy quark and anti-quark with a given background metric [11]. To obtain the heavy quark potential, we consider a Wilson loop living on the boundary of the five-dimensional AdS space where the heavy quark and anti-quark are set at $x = r/2$ and $x = -r/2$, respectively, see Fig. 1.

As a warm-up, we first work with the AdS black hole background in the Fefferman-Graham form. The background is given by, in Euclidean,

$$ds^2 = \frac{1}{z^2} \left(\frac{(1 - az^4)^2}{1 + az^4} dt^2 + (1 + az^4) d\vec{x}^2 + dz^2 \right), \quad (2)$$

where $a = (\pi T)^4/4$. We calculate the heavy-quark potential following [11]. The Nambu-Goto action for the open string connecting the boundary quark and the anti-quark is given by

$$S_{\text{NG}} = \frac{1}{2\pi\alpha} \int d^2\sigma \sqrt{\det G_{nm} \partial_\alpha X^n \partial_\beta X^m}. \quad (3)$$

After the gauge fixing,

$$\sigma^0 = t, \quad \sigma^1 = x \quad \text{and} \quad z = z(x), \quad (4)$$

the Nambu-Goto action on the background metric (2) becomes

$$S_{\text{AdSBH}} = \frac{1}{2\pi\alpha} \int_{-T/2}^{T/2} dt \int_{-r/2}^{r/2} dx \frac{1}{z^2} (1 - az^4) \sqrt{\frac{1 + az^4 + z'^2}{1 + az^4}}, \quad (5)$$

where $'$ means the derivative with respect to x , and T is the time interval. Considering the variable x as a time variable, the Hamiltonian of the system, which should be conserved, reads

$$\frac{1}{z^2} (1 - az^8) \frac{1}{\sqrt{(1 + az^4)(1 + az^4 + z'^2)}} = \text{const} = \frac{1}{z_0^2} (1 - az_0^4), \quad (6)$$

where $z_0 = z|_{x=0}$ with $z'|_{x=0} = 0$. From this equation, we find the relation between inter-quark distance r and z_0

$$r = 2 \int_0^{z_0} dz \frac{z^2}{z_0^2} \frac{1 - az_0^4}{1 - az^4} \frac{1}{\sqrt{1 + az^4}} \frac{1}{\sqrt{1 - \frac{z^4(1 - az_0^4)^2}{z_0^4(1 - az^4)^2}}}. \quad (7)$$

The regularized energy is

$$E_{\text{AdSBH}}^R \equiv \frac{S_{\text{AdSBH}}}{T} = \frac{1}{\pi\alpha'} \int_0^{z_0} dz \frac{1 - az^4}{z^2 \sqrt{1 + az^4}} \frac{1}{\sqrt{1 - \frac{z^4(1 - az_0^4)^2}{z_0^4(1 - az^4)^2}}} - \frac{1}{\pi\alpha'} \int_0^{z_h} dz \frac{1}{z^2} \frac{1 - az^4}{\sqrt{1 + az^4}}, \quad (8)$$

where $z_h = 1/a^{1/4}$, and the second term is corresponding to the masses of two free heavy quarks at finite temperature [11]. So Eq. (8) is nothing but the energy, equivalently potential, of heavy quark pair minus two free quark masses. Our results are shown in Fig. 2. Note that in the figure we plot the potential given in Eq. (8) only up to r^* , where the potential becomes zero, and for $r > r^*$ the potential in Eq. (8) becomes positive. The dissociation of heavy quarkonia happens when the potential becomes zero. As expected, at low temperature the heavy quarkonia are hard to dissociate since r^* is rather big, for instance $r^* = 0.93$ fm

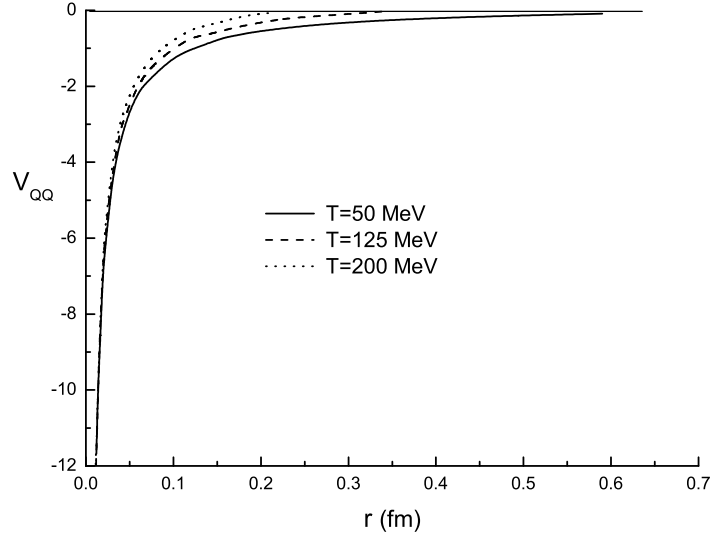


FIG. 2: The heavy quark potential on the AdS black hole. Here $V_{QQ} \equiv \pi\alpha' E_{\text{AdSBH}}^R$.

at $T=50$ MeV. As the temperature increases, r^* decreases, and so the dissociation is more likely to happen, for example, $r^* = 0.24$ fm at $T=200$ MeV. We remark here that to be more realistic, we should be able to distinguish the dissociation temperature of charmonium from that of bottomonium. For this we may need to introduce an energy scale such as an infrared cutoff other than the temperature to fix the scale of the system at hand.

III. GLUON CONDENSATION AND HEAVY QUARK POTENTIAL

To study the effect of the gluon condensation on the heavy quark potential, we consider the 5D gravity action in Euclidean with a dilaton coupled

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left(-\mathcal{R} - \frac{12}{R^2} + \frac{1}{2} \partial_M \phi \partial^M \phi \right), \quad (9)$$

where κ^2 is the 5D Newton constant, and R is the AdS curvature. By solving the coupled equations, dilaton equation of motion and the Einstein equation, with a suitable ansatz, we obtain two relevant backgrounds. The dilaton-wall solution is given by [12]

$$\begin{aligned} ds^2 &= \frac{R^2}{z^2} \left(\sqrt{1 - c^2 z^8} (d\vec{x}^2 + dt^2) + dz^2 \right), \\ \phi(z) &= \sqrt{\frac{3}{2}} \log \left(\frac{1 + cz^4}{1 - cz^4} \right) + \phi_0 \end{aligned} \quad (10)$$

where ϕ_0 is a constant and $c = 1/z_c^4$. Another one is the dilaton black hole solution [14, 15],

$$ds^2 = \frac{1}{z^2} \left(A d\vec{x}^2 + B dt^2 + dz^2 \right), \quad (11)$$

where

$$\begin{aligned} A &= (1 + fz^4)^{(f+a)/2f} (1 - fz^4)^{(f-a)/2f} \\ B &= (1 + fz^4)^{(f-3a)/2f} (1 - fz^4)^{(f+3a)/2f} \\ f^2 &= a^2 + c^2, \end{aligned} \quad (12)$$

and the corresponding dilaton profile is given by

$$\phi(z) = \phi_0 + \frac{c}{f} \sqrt{\frac{3}{2}} \log \left(\frac{1 + fz^4}{1 - fz^4} \right). \quad (13)$$

Here a is a temperature if $c = 0$. In both backgrounds, by expanding the dilaton profile near the boundary $z = 0$, we obtain

$$\phi(z) = \phi_0 + \sqrt{6}cz^4 + \dots, \quad (14)$$

and we can see that c is nothing but the gluon condensation up to a constant by the AdS/CFT dictionary. In this work, we take $\phi_0 = 0$.

As shown in [15], there is a Hawking-Page transition between the thermal dilaton-wall solution and the dilaton black hole solution at some critical value of a . Thus, the dilaton-wall background is for confined phase, and the dilaton black hole describes deconfined phase.

A. Dilaton wall background

First, we consider the Euclidean dilaton-wall background [12]

$$ds^2 = \frac{R^2}{z^2} \left(\sqrt{1 - c^2 z^8} (d\vec{x}^2 + dt^2) + dz^2 \right). \quad (15)$$

From now on we will take $R = 1$ for simplicity. With the gauge fixing

$$\sigma^0 = t, \quad \sigma^1 = x \quad \text{and} \quad z = z(x), \quad (16)$$

the Nambu-Goto action on the background metric (15) becomes

$$S_{\text{DAdS}} = \frac{1}{2\pi\alpha} \int_{-T/2}^{T/2} dt \int_{-r/2}^{r/2} dx e^{\frac{\phi}{2}} \frac{1}{z^2} \sqrt{1 - c^2 z^8 + \sqrt{1 - c^2 z^8} z'^2}. \quad (17)$$

The first integral, Hamiltonian, is then

$$H = -\frac{1}{2\pi\alpha} \frac{1 - c^2 z^8}{z^2 \sqrt{1 - c^2 z^8 + \sqrt{1 - c^2 z^8} z'^2}}. \quad (18)$$

Due to the conserved Hamiltonian, after comparing the Hamiltonian in (18) with that evaluated at $z = z_0$

$$H = -\frac{1}{2\pi\alpha} \frac{\sqrt{1 - c^2 z_0^8}}{z_0^2}, \quad (19)$$

we can easily find the integral relation between r and z_0

$$r = 2 \int_0^{z_0} dz \frac{z^2}{z_0^2} \frac{\sqrt{1 - c^2 z_0^8}}{(1 - c^2 z^8)^{1/4} \sqrt{\frac{\delta^2}{\delta_0^2} (1 - c^2 z^8) + c^2 z_0^4 z^4 - \frac{z^4}{z_0^4}}}, \quad (20)$$

where

$$\delta = \left(\frac{1 + cz^4}{1 - cz^4} \right)^{\sqrt{\frac{3}{8}}}, \quad \delta_0 = \left(\frac{1 + cz_0^4}{1 - cz_0^4} \right)^{\sqrt{\frac{3}{8}}}. \quad (21)$$

The value of c ($= 1/z_c^4$) could be determined as follows. As in the work of Csaki and Reece [12], the lightest glueball mass calculated on the dilaton wall background is $\sim 6.61/z_c$ which is compared with the lattice value ~ 1.73 GeV to fix the value of $1/z_c$: $c \sim (0.26 \text{ GeV})^4$. Another way to fix the value of z_c is to fit a heavy quarkonium mass. In [9], z_c is fixed by the lowest $c\bar{c}$ mass, 3.096 GeV, and it is given by $1/z_c = 1.29$ GeV. The regularized energy is given by

$$E_{\text{DAdS}}^R = \frac{1}{\pi\alpha'} \int_0^{z_0} dz \frac{\delta^2 (1 - c^2 z^8)^{1/4}}{\delta_0 z^2} \frac{(1 - c^2 z^8)^{1/2}}{\sqrt{\frac{\delta^2}{\delta_0^2} (1 - c^2 z^8) + c^2 z_0^4 z^4 - \frac{z^4}{z_0^4}}} - \frac{1}{\pi\alpha'} \int_0^{z_c} dz \delta \frac{(1 - c^2 z^8)^{1/4}}{z^2}, \quad (22)$$

where z_c behaves as an IR cutoff, so z and z_0 should be defined in the region smaller than z_c . We show r as a function of z_0 and plot the potential in Fig. 3.

B. Dilaton black hole background

Now, we move on to the dilaton black hole background,

$$ds^2 = \frac{1}{z^2} (A d\vec{x}^2 + B dt^2 + dz^2), \quad (23)$$

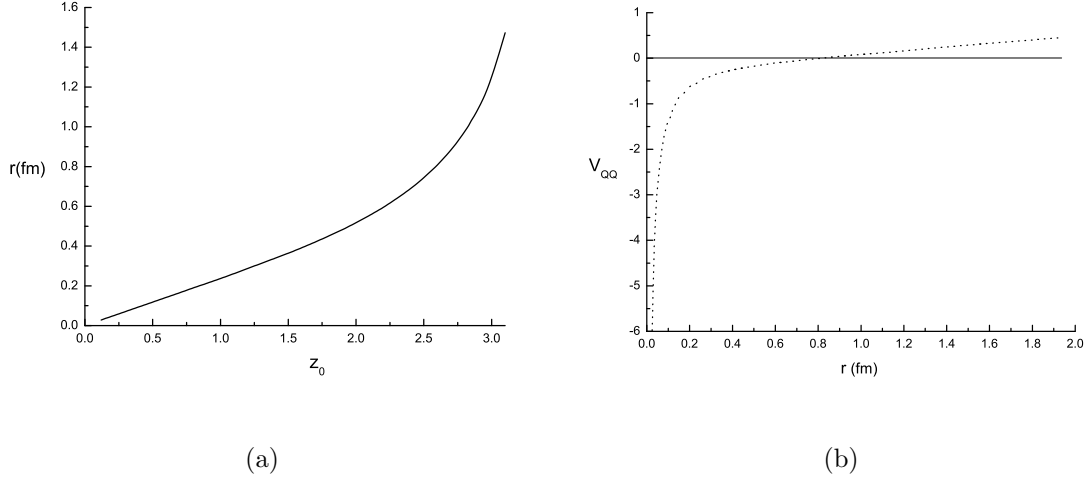


FIG. 3: (a) Inter-quark distance r as a function of z_0 with $1/z_c = 0.26$ GeV, where, and (b) the corresponding heavy quark potential.

where

$$\begin{aligned}
A &= (1 + fz^4)^{(f+a)/2f} (1 - fz^4)^{(f-a)/2f} \\
B &= (1 + fz^4)^{(f-3a)/2f} (1 - fz^4)^{(f+3a)/2f} \\
f^2 &= a^2 + c^2.
\end{aligned} \tag{24}$$

This dilaton black hole solution becomes the AdS black hole solution when $c = 0$, and it reduces to the dilaton wall background with $a = 0$. According to the Hawking-Page analysis done in [15], this metric is for deconfined phase. For discussion on the phases associated with the dilaton black hole background in other context, we refer to [14]. As discussed, c is the gluon condensation. Although the metric does not allow to define the Hawking temperature [13, 15] by requiring absence of conical singularity as long as $c \neq 0$, we could associate a with a temperature [15]. For more details about thermodynamics on the dilaton black hole background, we refer to [15]. On the dilaton black hole background, the Nambu-Goto action is given by

$$S_{\text{DAdSBH}} = \frac{T}{2\pi\alpha'} \int_{-r/2}^{r/2} dx e^{\frac{\phi}{2}} \frac{1}{z^2} \sqrt{AB + Bz'^2}, \tag{25}$$

where we used the same gauge fixing in (4). Then, the conserved Hamiltonian gives rise to the connection between z and x

$$z' = \frac{1}{z^2 \sqrt{B_0 A_0}} \sqrt{(z_0^4 B A - z^4 B_0 A_0) A}, \tag{26}$$

where $A_0 = A_{z=z_0}$ and $B_0 = B_{z=z_0}$. From this, we obtain a relation between r and z_0 .

$$r = 2 \int_0^{z_0} dz z^2 \sqrt{B_0 A_0} \frac{1}{\sqrt{(z_0^4 B A \frac{\gamma^2}{\gamma_0^2} - z^4 B_0 A_0) A}}, \quad (27)$$

where

$$\gamma = \left(\frac{A}{B}\right)^{\sqrt{\frac{3}{2}} \frac{c}{4a}}, \quad \gamma_0 = \left(\frac{A_0}{B_0}\right)^{\sqrt{\frac{3}{2}} \frac{c}{4a}}. \quad (28)$$

Hence, after using the similar regularization method in the previous section, we finally obtain the regularized energy on the dilaton black hole background

$$E_{\text{DAdSBH}}^R = \frac{1}{\pi \alpha'} \int_0^{z_0} dz \frac{\gamma^2}{\gamma_0} \frac{\sqrt{B}}{z^2} \frac{\sqrt{AB}}{\sqrt{\frac{\gamma^2}{\gamma_0^2} AB - A_0 B_0 \frac{z^4}{z_0^4}}} - \frac{1}{\pi \alpha'} \int_0^{z_f} dz \gamma \frac{\sqrt{B}}{z^2}, \quad (29)$$

where z_f is the position of the IR cutoff defined by $z_f = f^{-1/4}$.

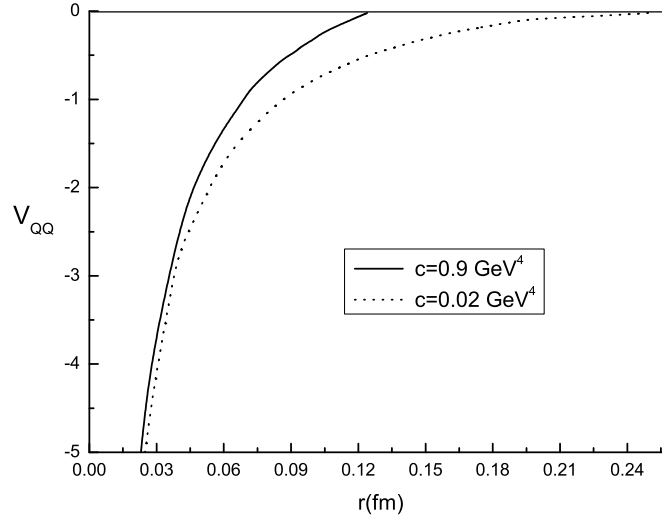


FIG. 4: The gluon condensate c dependence of the heavy quark potential with fixed a . Here $a = (\pi T)^4/4$, and we take $T = 200$ MeV.

In Fig. 4, we plot the heavy quark potential with a fixed a for a few values of c . From Fig. 4, we can see that the heavy quark potential becomes deeper as c decreases, implying the value of the gluon condensate drops in deconfined phase, the mass of heavy quarkonium decreases. For the heavy quarkonium, this is so since the mass of it is roughly equal to the

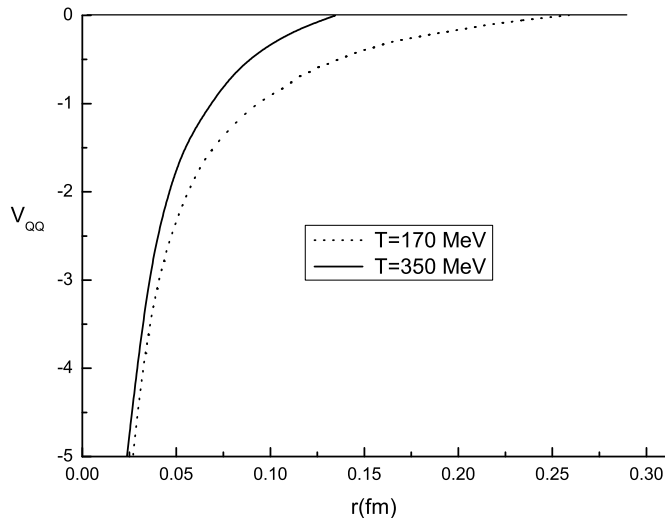


FIG. 5: The temperature dependence of the heavy quark potential with a fixed gluon condensate c . Here we take $c = 0.02 \text{ GeV}^4$.

masses of two heavy quarks plus the potential. Noting that the value of the gluon condensate suddenly drops near T_c of QCD deconfinement transition [7, 8], we may conclude that our study predicts dropping quarkonium mass just after the deconfinement transition.

Finally, we study the a dependence of the potential with fixed c . Note that a is related to a temperature, $a \sim T^4$, [15]. As shown in Fig. 5, the potential becomes shallow as a (temperature) increases. This means that the mass of heavy quarkonium increases as we increases the temperature, which is consistent with the result in [9]. As expected the value of r^* becomes smaller with increasing temperature, and so easy dissociation of heavy quarkonium at high temperature.

C. Heavy quarkonium and gluon condensate

Using the heavy quark potential obtained in section III B, we estimate the mass of heavy quarkonium by solving the Schrödinger equation, following potential model approach. In potential models, it is assumed that the interaction between two heavy quarks in a heavy quarkonium could be described by a heavy quark potential. Since the mass of heavy quarks are much larger than a typical scale of QCD, $m_{c,b} \gg \Lambda_{\text{QCD}}$, the heavy quark system is

generically non-relativistic.

Our aim in this section is not to give exact numbers for the masses, but to see the role of gluon condensate in the heavy-quark system. To this end we follow an approximate method given in [17] and consider only the leading order in [17] for simplicity. Here we briefly summarize the method and refer to [17] and references there in for details. To use the method, potentials should satisfy the following conditions:

$$\begin{aligned} V(r) &= -Ar^{-\alpha} + \kappa r^\beta + V_0, \quad \alpha, \beta > 0, \\ V'(r) &> 0 \quad \text{and} \quad V''(r) \leq 0 \end{aligned} \quad (30)$$

where V_0 is a constant, and A and κ are positive constants. We confirmed that our potential in section III B satisfies these requirements. Then the binding energy is given by

$$E_{n,l} \simeq V(r_0) + \frac{1}{2}r_0V'(r_0), \quad (31)$$

where higher orders are neglected for simplicity. r_0 is determined by

$$1 + 2l + (2n_r + 1) \left[3 + \frac{r_0 V''(r_0)}{V'(r_0)} \right]^{1/2} = \left[8\mu r_0^3 V'(r_0) \right]^{1/2}, \quad n_r, l = 0, 1, 2, 3, \dots \quad (32)$$

Finally, we arrive at the mass of bound state:

$$M_{\bar{Q}Q} = 2m_Q + 2E_{n,l}. \quad (33)$$

Now, as examples, we consider charm and bottom quarks.

c (GeV ⁴)	r_0 (GeV ⁻¹)	binding energy (GeV)
0.001	0.927	0.208
0.007	0.937	0.217
0.02	0.982	0.248

Table 1. Charm-quark system and gluon condensate. Here the charm quark mass is 1.396 GeV and $T = 200$ MeV.

In Table 1, we consider the charm-quark system. Since the binding energy is positive, we have no bound state out of charm quarks. Here we don't claim that there is no charmonium in QGP since we adopted a very crude approximation. What is interesting here is that smallness

of gluon condensate supports the existence of charmonium since the positive binding energy decreases with decreasing gluon condensate c . This is, as it should be, consistent with our observation made in the previous section: decreasing c means the deepening in the potential. But, we have no clear physical interpretation on this observation.

c (GeV ⁴)	r_0 (GeV ⁻¹)	binding energy (GeV)	meson mass (GeV)
0.02	0.285	-0.624	8.357
0.2	0.291	-0.465	8.675
0.9	0.304	-0.121	9.364

Table 2. Bottom-quark system and gluon condensate. Here the charm quark mass is 4.803 GeV and $T = 200$ MeV

In Table 3, we consider the bottom-quark system. As expected, the mass of bottomonium decreases as the gluon condensate is to be smaller.

IV. SUMMARY AND DISCUSSION

With an observation [8] that the gluon condensate is an useful quantity to characterize the QCD deconfinement transition, we calculate the holographic heavy quark potential on deformed AdS backgrounds with gluon condensate included. We also solved the Schrödinger equation approximately to estimate the mass of heavy quarkonium, following the potential model approach.

Our analysis of the dilaton black hole metric with fixed a reveals that the potential becomes deeper as the gluon condensate decreases in deconfined phase. We associate this with the dropping of the heavy quarkonium mass in deconfined phase. With the deconfinement transition, the value of the gluon condensate drops much, and therefore the dropping mass is most likely to occur just after the deconfinement transition. Finally, we study the a dependence of the heavy quark potential and show that the potential becomes shallow as we increase a . If we interpret a as a temperature [15], our result indicates that the mass of the heavy quarkonium increases with temperature in deconfined phase.

As we increase the temperature right after the deconfinement transition, the mass of a heavy quarkonium increases with temperature, while it decreases with a decreasing gluon

condensate. Just after the deconfinement transition, although the gluon condensate is not an order parameter of the deconfinement transition, its value does change dramatically with the deconfinement transition. Even though the deconfinement transition is driven by the temperature, the drastic drop in the value of the gluon condensate is from the deconfinement transition itself since the value of the temperature changes very little around the transition point. Therefore, right after the deconfinement transition, the effect of the gluon condensate should dominate over the temperature.

In conclusion, based on the drastic drop of the gluon condensate right after the deconfinement transition observed in lattice QCD [8], our study predicts that the mass of heavy quarkonium just after the deconfinement transition decreases with decreasing gluon condensate, which is consistent with [7, 9]. As we increase the temperature further above T_c , gluon condensate reduces to increase the mass of heavy quarkonium, while increasing T will make it heavier. Therefore, to reach a concrete conclusion on the mass of heavy quarkonium at high temperature, except very close to T_c , we have to perform detailed study on the competition of the two effects: effects of the gluon condensate and temperature (pure thermal). To this end, it is essential to calculate the temperature dependence of the gluon condensate in AdS/CFT.

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